Unit Conversions 2 – Scientific Notation and Significant Digits

[Note to teachers and students: This lesson is designed to be a follow-up to the Unit Conversions Student Exploration sheet. The same Gizmo is used for both activities.]

**Vocabulary:** resolution, scientific notation, significant digits

**Prior Knowledge Questions** (Do these BEFORE using the Gizmo.)

[Note: The purpose of these questions is to activate prior knowledge and get students thinking. Students are not expected to know the answers to the Prior Knowledge Questions.]

Philip measures a room using his feet. (His feet are each about a foot long.) He estimates the room measures about ten and a half feet by thirteen and a half feet. He calculates the room’s area to be: 10.5’ × 13.5’ = 141.75 ft².

1. Which do you think is the best description of the area of the room? (Circle your choice.)
   - A. The room’s area is exactly 141.75 ft².
   - B. The room’s area is about 142 ft².

2. Explain your choice: *Explanations will vary.*

   Sample answer: I chose B because he used his feet to roughly estimate the dimensions of the room, so it is unlikely he could calculate the area with precision.

**Gizmo Warm-up**

When scientists report a value, they have to take several things into consideration. Sometimes values are very large or small. In this case, scientists can use shorthand called **scientific notation** to report the value. Scientists also must consider how precise their value is. The precision of a measurement can be shown by the number of **significant digits** in the value.

To begin, check that the Burj Khalifa question is shown. Drag the three tiles shown below to determine the tower’s height in micrometers. The answer is given in scientific notation.

1. The Burj Khalifa is 828,000,000 micrometers tall. How is this value written in scientific notation in the Gizmo™?
   - 8.28 • 10⁸ micrometers

2. In a value, any non-zero digit is considered a significant digit. (Zeroes may or may not be significant.) What is the minimum number of significant digits in 828,000,000?
   - 3
Activity A: Scientific notation

Get the Gizmo ready:
- Select **Metric units only** and **Distance** from the **Conversion** menu. Make sure **Show result** is off.
- Click **Next** until you reach the question about Proxima Centauri.

Question: How can you convert numbers into and out of scientific notation?

1. **Observe**: Some of the problems in this Gizmo involve very small or very large quantities. Look at the bottom three **Unit Conversion Tiles**. What do you notice in the numerator?

   **Sample answer: Each numerator is a product of a number and a power of 10.**

   The numbers in the numerators are written in scientific notation. In scientific notation, a number is converted to the product of a number between 1 and 10 and a power of 10. For example, 1,000,000 is written as $1.0 \times 10^6$. The first part of this number is called the **coefficient**. The second part is called the **base**.

2. **Convert**: To convert a number written in scientific notation into a standard number, first look at the exponent on the base. If it is positive, move the decimal point on the coefficient to the right as many times as the exponent indicates, as shown below:

<table>
<thead>
<tr>
<th>Look at exponent</th>
<th>Count digits</th>
<th>Move decimal point</th>
<th>Standard form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$8.35 \times 10^7$</td>
<td>1 2 3 4 5 6 7</td>
<td>83 500 000</td>
<td>83,500,000</td>
</tr>
</tbody>
</table>

   Practice converting the two numbers below into standard form:

   $1.0 \times 10^9 = 1,000,000,000$  $6.72 \times 10^{12} = 6,720,000,000,000$

   You can perform this process in reverse to convert numbers in standard form into scientific notation. The number of times you move the decimal point to the left will be equal to the exponent on your base. Remember there should be only one digit to the left of the decimal point in scientific notation.

<table>
<thead>
<tr>
<th>Standard form</th>
<th>Place new decimal point</th>
<th>Count digits</th>
<th>Scientific notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>3,700,000</td>
<td>3.700 000</td>
<td>1 2 3 4 5 6</td>
<td>$3.7 \times 10^6$</td>
</tr>
</tbody>
</table>

   Practice this with the two numbers below:

   $8,200,000 = 8.2 \times 10^6$  $50,880,000,000,000 = 5.088 \times 10^{13}$

   *(Activity A continued on next page)*
Activity A (continued from previous page)

3. Solve: Look at the last tile.
   A. How many kilometers are equal to 1 light year? \(9.461 \times 10^{12} \text{ km}\)
   B. Write this number in standard form: \(9,461,000,000,000 \text{ km}\)
   C. Drag this tile below to solve the problem. Turn on Show result. What is the distance to Proxima Centauri in kilometers? \(4.013 \times 10^{13} \text{ km}\)
   D. Write this distance in standard form: \(40,130,000,000,000 \text{ km}\)

4. Convert: Not all numbers written in scientific notation are very large numbers. Scientific notation also can be used to write very small numbers. This is done by making the exponent on the base negative, indicating the decimal point should be moved to the left.

<table>
<thead>
<tr>
<th>Look at exponent</th>
<th>Count digits</th>
<th>Move decimal point</th>
<th>Standard form</th>
</tr>
</thead>
<tbody>
<tr>
<td>(7.9 \times 10^{-6})</td>
<td>123 456</td>
<td>0.000 07.9</td>
<td>0.0000079</td>
</tr>
</tbody>
</table>

Try converting these numbers into standard form:

\(1.0 \times 10^{-10} = 0.0000000001\) \(1.6 \times 10^{-7} = 0.00000016\)

You can perform this process in reverse to convert numbers in standard form into scientific notation, as shown below.

<table>
<thead>
<tr>
<th>Standard form</th>
<th>Place new decimal point</th>
<th>Count digits</th>
<th>Scientific notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000 05</td>
<td>0.000 05.0</td>
<td>123 45</td>
<td>5.0 \times 10^{-5}</td>
</tr>
</tbody>
</table>

Practice this with the two numbers below:

\(0.00012 = 1.2 \times 10^{-4}\) \(0.00000000458 = 4.58 \times 10^{-9}\)

5. Practice: Click Next so that you see the question about a helium atom.
   A. What is the diameter of a helium atom in meters? \(9.8 \times 10^{-11} \text{ meters}\)
   B. Write this number in standard form: \(0.000000000098 \text{ meters}\)
Introduction: When you report a measurement, you are really doing two things. Not only are you communicating the value of the measurement, but you are also stating something about the resolution, or fineness, of your measurement. This is conveyed by the number of significant digits in the value.

Question: What digits are significant, and why?

1. Think about it: Mark measures the volume of water in a beaker marked in 50-mL intervals. He reports the volume is 43.927 mL. What is misleading about this value?

   Sample answer: The reported volume is very precise, and implies that Mark used a very fine instrument that can measure volume to the nearest thousandth of a milliliter, which is not true. Instead, Mark is estimating the volume based on a very imprecise instrument.

Ideally, reported measurements should reflect the resolution of the instrument used to obtain the data. With Mark's beaker, a better value might be “about 50-mL.” This is done by specifying the number of significant digits in the data. A significant digit is any value that has been directly measured. Other digits are not significant.

Here are some rules for determining whether a digit is significant:

**Significant Digit Rules and Examples**

1. Any non-zero digit is significant: 227.4 has four significant digits.

2. Any digit that is between other significant digits is significant: 200.08 has five significant digits.

3. Zeros to the right of a significant digit and to the right of a decimal point are always significant: 6.00 has three significant digits.

4. Zeros used to space a number to the right of a decimal point are not significant: 0.000147 has only three significant digits.

5. In scientific notation, all digits in the coefficient are significant: 8.75 x 10^5 has three significant digits.

6. Zeros to the right of a significant digit but to the left of a decimal point may or may not be significant: 875,000 has at least three significant digits, but may have as many as six.

7. If a number ends in a decimal point, the zeros to the left of the decimal point are significant: 875,000. has six significant digits.

(Activity B continued on next page)
Activity B (continued from previous page)

2. **Practice**: Look at the quantities mentioned in the **Metric units only Distance** problems. For each value, state the number of significant digits in the first column and the rule(s) you used to determine the number of significant digits. The first has been done for you.

<table>
<thead>
<tr>
<th>Question</th>
<th>Number of significant digits</th>
<th>Rule(s) used</th>
</tr>
</thead>
<tbody>
<tr>
<td>The tallest building in the world, the Burj Khalifa in Dubai, is 0.828 kilometers high.</td>
<td>3</td>
<td>1, 4</td>
</tr>
<tr>
<td>The largest human cell is the egg cell, with a diameter of 121 micrometers.</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>On a caterpillar’s map, all distances are marked in millimeters. The caterpillar’s map shows that the distance between two milkweed plants is 4,012 millimeters.</td>
<td>4</td>
<td>1, 2</td>
</tr>
<tr>
<td>The closest star to our Sun is Proxima Centauri, which is 4.242 light years away.</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>A helium atom has a diameter of $9.8 \times 10^{-11}$ meters.</td>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

3. **Practice**: The following quantities are all found in the **Speed** problems (both metric and mixed units). State the number of significant digits for each value and the rule(s) you used to determine the number of significant digits.

<table>
<thead>
<tr>
<th>Metric units only</th>
<th></th>
<th>Mixed units</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Value</strong></td>
<td><strong>Significant digits</strong></td>
<td><strong>Rule(s)</strong></td>
<td><strong>Value</strong></td>
</tr>
<tr>
<td>3.50 m/s</td>
<td>3</td>
<td>1, 3</td>
<td>113 km/hr</td>
</tr>
<tr>
<td>72 km/hr</td>
<td>2</td>
<td>1</td>
<td>1.25 mi</td>
</tr>
<tr>
<td>4.25 min</td>
<td>3</td>
<td>1</td>
<td>2.00 min</td>
</tr>
<tr>
<td>$2.0 \times 10^3$ μm/s</td>
<td>2</td>
<td>5</td>
<td>9.58 s</td>
</tr>
<tr>
<td>45.215 d</td>
<td>5</td>
<td>1</td>
<td>1.0 yd/d</td>
</tr>
<tr>
<td>60,004,000 km</td>
<td>5-8</td>
<td>1, 2, 6</td>
<td>1.02 mm/hr</td>
</tr>
</tbody>
</table>
**Activity C:**
Calculating with significant digits

<table>
<thead>
<tr>
<th>Get the Gizmo ready:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Select <strong>Speed</strong> and <strong>Metric units only</strong>.</td>
</tr>
<tr>
<td>• Turn off <strong>Show result</strong>.</td>
</tr>
<tr>
<td>• If necessary, click <strong>Next</strong> until you reach the question about Marcia.</td>
</tr>
</tbody>
</table>

**Question:** If you multiply or divide two numbers, how many significant digits are in the answer?

1. **Observe:** The first metric speed question is about Marcia, who ran at a speed of 3.50 meters per second. How many significant digits are in this value? **3**

2. **Solve:** Use the tiles to convert Marcia’s speed to kilometers per hour. When you are finished, turn on **Show result** and click **Submit** to check if the answer is correct.
   
   A. What value do you get? **12.6 kilometers/hour**
   B. How many significant digits are in this value? **3**

3. **Learn a rule:** In any calculation, the number of significant digits in the answer should not be greater than the number of significant digits of any measured value. For example, suppose you do the following area calculation:

   \[ 5.73 \text{ cm} \times 2.1 \text{ cm} = 12.033 \text{ cm}^2 \]

   In this situation, because the second measurement has the fewest significant digits, the answer must also have just two significant digits and should be reported as 12 cm².

   For values that are exact, there are unlimited significant digits. For example, there are exactly 100 centimeters in a meter. For conversion factors that are exact, you do not have to worry about the significant figures in the conversion factor.

   The list below shows all the conversion factors in the Gizmo that are not exact values. For each inexact conversion factor, write the number of significant digits it has:

   - 9.461 \times 10^{12} \text{ km/1 light year} \hspace{1cm} 4
   - 2.20462 \text{ pounds/1 kilogram} \hspace{1cm} 6
   - 1.60934 \text{ kilometers/1 mile} \hspace{1cm} 6
   - 16.3871 \text{ cm}^3/1 \text{ cubic inch} \hspace{1cm} 6
   - 453.6 \text{ grams/1 pound} \hspace{1cm} 4
   - 3.785 \text{ liters/1 gallon} \hspace{1cm} 4
   - 28.35 \text{ grams/1 ounce} \hspace{1cm} 4
   - 1.308 \text{ cubic yards/1 m}^3 \hspace{1cm} 4

   (Activity C continued on next page)
Activity C (continued from previous page)

4. **Practice**: Turn off **Show results** and click **Next**. For each of the remaining metric speed problems, predict the number of significant digits in the answer. Then, solve each problem and write the answer value and number of significant digits. Turn on **Show results** to check your answers.

   A. A tuna travels at a speed of 72 kilometers per hour. What is the speed of the tuna in meters per second?

      Predicted number of significant digits in answer: 2

      Answer value: 20. Actual number of significant digits: 2

      [Note: This is an example of a number that ends in a decimal point, indicating that the zero to the left of the decimal point is significant.]

   B. A slug takes 4.25 minutes to travel 11.2 centimeters. What is the speed of the slug in meters per second?

      Predicted number of significant digits in answer: 3

      Answer value: $4.39 \times 10^{-4}$ m/s Actual number of significant digits: 3

   C. A paramecium is a single-celled organism that lives in ponds. It travels at a rate of 2,000 micrometers per second. What is the speed of the paramecium in meters per hour? (Hint: Look at the first tile to see how many significant digits are in the value.)

      Predicted number of significant digits in answer: 2

      Answer value: 7.2 Actual number of significant digits: 2

      [Note: In this problem, the number 2,000 in the question has an ambiguous number of significant digits. The actual significant digits are shown by the first tile, which is written in scientific notation $2.0 \times 10^3$ micrometers/second.]

   D. A space probe takes 45.215 days to travel to Mars, a distance of 60,004,000 kilometers. What is the speed of the probe in meters per second?

      Predicted number of significant digits in answer: 5

      Answer value: $15,360.\text{ meters/second}$ Actual number of significant digits: 5

5. **Think and discuss**: Why is it important to pay attention to significant digits in science and engineering?

   Sample answer: A measurement should indicate not only a value, but also give the reader a sense of how precise the measurement is. A number written with too many significant digits can give the reader the impression that the value is more precise than it really is.